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ARGUMENTOS

Miguel López-Astorga*

Hempel's paradox is not an actual cognitive problem

ABSTRACT

The paradox of the black ravens proposed by Carl Hempel is a problem if it is analyzed from standard logic. This is so because the equivalences in that logic can lead one to think that individuals should carry out certain actions that they do not usually do. In this paper, I show that, if we assume frameworks other than standard logic, the problem disappears, since those same actions are not expected. In particular, I use as examples two approaches: that of Stoic logic and that of the mental models theory.

Keywords: Hempel's paradox; mental models; reasoning; standard logic; Stoic logic.

RESUMO

O paradoxo dos corvos negros proposto por Carl Hempel é um problema se for analisado a partir da lógica padrão. Isto é assim porque as equivalências nessa lógica pode levar a pensar que os indivíduos devem realizar determinadas ações que eles não costumam fazer. Neste artigo, vou mostrar que, se ignorarmos a lógica padrão e assumirmos outros sistemas, o problema desaparece, uma vez que essas mesmas ações não são esperadas. Em particular, eu uso como exemplos duas abordagens: a da lógica estoica e a da teoria dos modelos mentais.

Palavras-chave: paradoxo de Hempel; modelos mentais; raciocínio; lógica padrão; lógica estoica.

* Ph.D. in Logic and Philosophy of Science (University of Cádiz, Spain). Professor and researcher at the Institute of Humanistic Studies "Juan Ignacio Molina," University of Talca, Chile. E-mail: milopez@utalca.cl

Introduction

Hempel (1945) presented an interesting paradox that appears to continue to be a problem in our days. That paradox is known as the paradox of the black ravens and is based on a universal affirmative sentence, that is, a sentence of the kind 'all of the P are Q.' Several papers have addressed the difficulties related to this paradox, but I will mainly focus on that of López-Astorga (2008) and that of Nickerson (1996). As the latter (NICKERSON, 1996, p. 2) reminds us, the basic challenge of the paradox refers to the fact that, as indicated by Hempel, a sentence such as 'all ravens are black' is equivalent to another sentence such as 'all nonblack things are nonravens.' This is so because, as it is well known, in standard first-order predicate calculus the following two formulae are equivalent:

[I]: (x) (Px -> Qx)

[II]: (x) (¬Qx -> ¬Px)

Where the brackets mean that x is universally quantified, P and Q are predicates, '->' stands for conditional relationship, and '¬' is the denial.

Thus, if [P] is interpreted as 'to be a raven' and [Q] as 'to be black,' we can easily note that, indeed, Hempel and Nickerson are right. But this equivalence raises an important cognitive problem. Given that, in first-order predicate logic, [I] can be translated into [II] and vice versa, if we were asked for checking whether or not the sentence 'all ravens are black' is true, we could, by virtue of [I], review ravens in order to confirm that they are black, and, by virtue of [II], review things with colors other than black in order to confirm that they are not ravens. From this point of view, because it is a nonblack thing, even a white shoe should support the idea that 'all ravens are black' (NICKERSON, 1996, p. 2). However, it is hard to accept facts such as the latter, and it is difficult to imagine an individual that, as a response to the need to verify whether or not all of the ravens are black, looked for different nonblack objects to this end.

Therefore, it seems that, as said, Hempel's paradox reveals a relevant problem that needs to be solved. Nevertheless, in my view, the problem is only apparent and not real. Its difficulties are related to the implicit assumption that the human mind is led by standard logic. If we do not accept that assumption, the problem fades. In this way, a very simple option to eliminate the difficulties linked to the paradox of the ravens can be to reject the thesis that standard logic describes human reasoning and to assume another theoretical framework. And I am saying that this one is a simple option because it is not easy to find at present a cognitive theory holding a direct relationship between the human inferential activity and standard calculus, either propositional calculus or first-order predicate calculus.

In any case, the main goal of this paper is to show that, true, there are other approaches from which Hempel's paradox is not a problem. In this way, I will resort to two frameworks that I will use as examples. One of them is ancient and comes from Hellenistic Greece: Stoic logic. The other one is very current and comes from the cognitive field science: the mental models theory. As it can be

seen below, in neither of these two cases the paradox is problematic. I begin with the analysis of it from Stoic logic and then continue with that from the mental models theory.

Stoic logic and the paradox of the black ravens

Actually, it is not necessary to explain here all of the theses of Stoic logic, but only those that are related to the difficulties of the paradox. It is true that, if we only take into account the passages that are kept and what the ancient sources literally state, it can be thought that Stoic logic did not pay much attention to quantified sentences, but mainly to sentences that today we could say that they correspond to propositional logic. Nonetheless, given that both [I] and [II] provides a conditional relationship and Stoic logic offers a clear account of the conditional, it can also be thought that this later logic has the theoretical machinery to deal with Hempel's paradox.

Thus, a very important point in Stoic logic is that a conditional is not that by the only fact that the term *εἰ* (if) is embedded in the sentence. According to Chrysippus of Soli, it is necessary that another condition is fulfilled too: the contrary (*ἀντικείμενον*) of the second clause, i.e., of the consequent, should be inconsistent with (*μάχεται*) the first clause, i.e., with the antecedent (e.g., CICERO, *De Fato* 12-16; DIOGENES LAËRTIUS, *Vitae Philosophorum* 7, 73; SEXTUS EMPIRICUS, *Pyrrhoniae Hypotyposes* 2, 111; BARNES, BOBZIEN, & MIGNUCCI, 2008, p. 107; GOULD, 1970, p. 76; LÓPEZ-ASTORGA, 2015, p. 9; O'TOOLE & JENNINGS, 2004, p. 492).

In this way, it seems that Chrysippus claimed a meaning connection between the two clauses, and that, if this connection could not be observed, the sentence was not a real conditional. But a very relevant argument in this regard is that given by López-Astorga (2015, p. 9). According to it, in Stoic logic the actual conditionals not only referred to the relationship [p -> q], but also to [¬q -> ¬p]. However, this only happened, as said, when the conditional was a true conditional, i.e., when it followed Chrysippus' criterion. If that criterion was not fulfilled, the sentence could not be related to [¬q -> ¬p], whether or not the word *εἰ* was included in it.

lthough expressed in terms of propositional logic, it is obvious that there is a clear correspondence between [p -> q] and [I] and between [¬q -> ¬p] and [II]. So, from the Stoic logic point of view, it could be said that, unlike standard logic, the equivalence between [I] and [II] is not always guaranteed. That equivalence is only possible when there is a connection between the antecedent and the consequent such that the opposite of the latter is impossible when the former is true. Nevertheless, in the case of the paradox of the black ravens, it is evident that the concept corresponding to the consequent is 'black' and the one corresponding to the antecedent is `raven.' And it is also absolutely clear that the contrary of 'black' (i.e., any color other than black) is not in conflict with 'raven,' since, if 'all ravens are black' needs to be confirmed, it is not known for sure which the color of the ravens is, and, therefore, no color other than black can be incompatible with the concept 'raven.' Thus, in Stoic logic sentences such as [I] and [II] may not be equivalent, which means that the paradox does not exist in this framework.

Of course, it could be argued against this that the Stoics distinguished two types of conditional (*συνημμένον*), and that the conditional fulfilling Chrysippus' requirement was only one of those types. Sedley (1984, p. 312) describes the other one, which is called '*ϖιθανὸν συνημμένον*' (very credible conditional). It is a conditional in which the connection criterion is not sure to be fulfilled, but it is very plausible that that is so. In these cases, what is appropriate is to express the conditional not as a conditional, but as a negated conjunction in which the second conjunct is negated as well, that is, not as [p -> q], but as {[¬(p · ¬q)], where '·' is conjunction}. Thus, given that one of the *ἀναϖόδεικτοι* (*indemonstrables*) that, according to Diogenes Laërtius (*Vitae Philosophorum* 7, 80), was proposed by Chrysippus of Soli was *Modus Ponendo Tollens I* {[¬(p · q)]; [p] / Ergo [¬q]}, the problem would remain. Indeed, *Modus Ponendo Tollens I* would allow drawing [q] from [¬(p · ¬q)] and [p], and [¬p] from [¬(p · ¬q)] and [¬q]. Therefore, if was assumed that [p] refers to 'raven' and [q] to 'black,' the situation would not be very different from that in which the conditional is not *ϖιθανόν*, but real. And this is so because any thing that was not black [¬q] would have to lead to conclude that that same thing is not a raven [¬p].

Nonetheless, in my view, this objection would not be adequate. While the correspondences between [p -> q] and [I] and between [¬q -> ¬p] and [II] can be justified in Stoic logic, it appears that this later logic does enable to think about quantified universally denied conjunctions. Indeed, Sedley's thesis seems to be that a *ϖιθανὸν συνημμένον* only refers to particular individuals or objects. In his paper, he analyzes *σωρίτης* (*sorites*) paradox and offers arguments such as that "[…] transitive properties, such as necessity, are not transmitted through a chain of negated conjunctions in the way that they are through a chain of strong conditionals. Now another transitive property is cognitive certainty" (SEDLEY, 1984, p. 313). Thus, the idea appears to be that, when we have confirmed only particular cases (although these are a significant number), we cannot use the conditional, but denied conjunctions. However, these denied conjunctions cannot link, as said, predicates under the scope of a universal quantifier, but only predicates that can be attributed to the same individual or object. So, we cannot state, for example,

$(x)\ \neg(Px \cdot \neg Qx)$

But only

$\neg(Pa_1 \cdot \neg Qa_1)$
$\neg(Pa_2 \cdot \neg Qa_2)$
$\neg(Pa_3 \cdot \neg Qa_3)$
…
$\neg(Pa_n \cdot \neg Qa_n)$

Where, in the case of the paradox of the black ravens, 'a_1', 'a_2', 'a_3',…, 'a_n' represent particular ravens.

In this way, to conclude, for example, [$\neg Pa_n$] from [$\neg(Pa_n \cdot \neg Qa_n)$] by means of Modus Ponendo Tollens I, it would be necessary to have the datum [$\neg Qa_n$], i.e., 'the raven [a_n] is not black.' But this is not possible, since [a_n], as well as [a_1], [a_2], [a_3],…, stands for a particular raven that has been already reviewed and is known

to be black. Therefore, it we already know that [a_n] is a raven [Pa_n] and that it is black [Qa_n], the simple supposition that [a_n] is not black [$\neg Qa_n$] makes no sense.

Therefore, it can be said that the paradox does not exist in Stoic logic. In this logic, given the sentence 'all ravens are black,' if we want to check whether or not it is true, it is not the same to review ravens in order to check whether or not they are black and to review things that are not black in order to check whether or not they are ravens. In that system, 'all ravens are black' is not equivalent to 'all nonblack things are nonravens.' Likewise, if we have not reviewed all of the ravens around the world, we cannot say, for example, 'it is not the case that a thing is a raven and that same thing is not black.' At most, we can state that 'it is not the case that this particular thing is a raven and this same thing is not black,' where the particular thing is a raven already known. This later sentence can be repeated several times being applied to different known ravens, but, as explained, it will be never possible to derive from those sentences the conclusion that every nonblack thing is not a raven, since, as argued, in Stoic logic each denied conjunction only refers to a particular individual or thing.

Nevertheless, as also indicated, this ancient logic is not the only system in which Hempel's paradox is not a problem. This is not so in a current cognitive theory either, the mental models theory. I show that in the next section.

The mental models theory and the paradox of the black ravens

True, the paradox is also no problem if the theoretical framework of the mental models theory is assumed. This theory has been explained, described, and analyzed in many papers and works (e.g., JOHNSON-LAIRD, 2010, 2012, 2015; KHEMLANI, LOTSTEIN, TRAFTON, & JOHNSON-LAIRD, 2015; LÓPEZ-ASTORGA, 2013, 2014; OAKHIL & GARNHAM, 1996), but only a very particular aspect of it is interesting for this paper: the way it deals with universally quantified affirmative sentences, i.e., with sentences such as [I].

In general, the theory claims that human reasoning processes are essentially semantic and that they basically consist of reviews of combinations of possibilities. Nonetheless, sometimes it is very hard to find all of the possible combinations, and, for this reason, the mental models theory often distinguishes the combinations that are easy to detect from the combinations that are difficult to identify. In the case of the sentences similar to [I], as it can be seen in Khemlani *et al*. (2015), the former are called 'canonical models,' and the name of the latter is 'noncanonical models.'

In this way, what is actually important to this paper is that, according to Khemlani *et al*. (2015, p. 2077, Table 1), an example of canonical model for [I] could be this one:

P	Q
P	Q
P	Q

Khemlani *et al*. (2015) do not use these letters {[P] and [Q]}, but [A] and [B]. However, what is relevant is that, in the case of the paradox of the black ravens, this model represents these three possible scenarios:

Raven	Black
Raven	Black
Raven	Black

This means that firstly individuals only think about, more or less, three possibilities: three cases in which the object is both a raven and black. Thus, if individuals do not make further effort, they only consider cases of ravens that should be black and do not pay attention to objects or things that are not black. And this is so because there are not objects or things with color other than black in any of the three possibilities of the later canonical model.

But after more reflection, the individual can note that there are also other possibilities and take into account a noncanonical model such as the following:

P	Q
¬P	Q
¬P	¬Q

In fact, with other letters too, this is the example of noncanonical model for sentences such as [I] offered by Khemlani *et al*. (2015, p. 2077, Table 1). And, in the case of Hempel's paradox, this means that a noncanonical model could lead to these three possible scenarios:

Raven	Black
¬(Raven)	Black
¬(Raven)	¬(Black)

So, as it can be noted, if only a canonical model is considered, there are only scenarios in which there are ravens. Only if a noncanonical model is taken into account, which requires reflection effort, one might realize that there can also be scenarios with nonblack things (the third one of the previous noncanonical model), and that, in those scenarios, those nonblack things cannot be ravens in any way. Indeed, in the noncanonical model, the only scenario with a nonblack thing (as said, the third one) is a scenario in which that thing is not a raven.

Therefore, it can be stated that, if the mental models theory is right, the reason why people, when they need to check whether a sentence such as 'all ravens are black' is true or false, tend to prefer to review ravens in order to verify that they are black to review nonblack things in order to verify that they are not ravens is absolutely obvious. The former option is clear with only paying attention to canonical models, which do not require effort. We can only be aware that it would be worth checking nonblack objects with more reflection and with a noncanonical model showing that, in the scenarios in which a thing is not black, that thing cannot be a raven. But the point is that, in everyday life, human beings often reason quickly and without great analyses, i.e., considering only canonical models.

Conclusions

This paper confirms something indicated above. The paradox of the black ravens is only a problem if we assume that the human mind works following the

rules, requirements, and equivalences of standard logic. If we adopt other logics or frameworks, all of the difficulties of the paradox can fade.

I have shown two examples in this regard here. One of them is an ancient approach and is linked to Hellenistic philosophy: Stoic logic. According to it, a sentence such as 'all ravens are black' is not actually a conditional. It is not even a weak (*ϖιθανόν*) one. So, the correspondence between [p -> q] and [¬q -> ¬p], or, if preferred, in terms of first-order predicate logic (which, of course, are not the most appropriate terms in Stoic logic), between [I] and [II], cannot be taken into account in this case and there is no a real paradox.

The other one is based on a contemporary theory, the mental models theory, describing human reasoning. Following this theory, the key is that the human mind works in such a way as to tend to ignore the equivalence between [I] and [II], at least, in the first instance. Our mind usually only considers canonical models and, to note the equivalence, further reflection is needed. That is why to look for ravens to check whether or not they are black often seems more natural and appropriate than to look for nonblack objects to check whether or not they are ravens.

The problem is, therefore, as said, standard logic. Really, this is not a new idea or finding. In fact, the literature on the mental models theory shows many examples of cases that appear to confirm that people do not always follow that logic. Besides, even the contemporary theories holding that there is a certain formal logic in our mind leading our inferential activities, and that individuals apply reasoning syntactic schemata, such as, for example, the mental logic theory (e.g., BRAINE & O'BRIEN, 1998; O'BRIEN, 2009, 2014; O'BRIEN & LI, 2013; O'BRIEN & MANFRINATI, 2010), recognize that standard logic cannot be the criterion to account for the conclusions that are often deduced from a set of premises, and that, in addition, this later logic includes a number of rules and principles that are many times ignored by people. So, it seems that it can be stated that the paradox of the black ravens is not an actual paradox, or at least that it is not a real paradox from a cognitive point of view. It is only so if the approach adopted is standard logic. But, if we assume that this logic is not the reference framework of human thought (and, as said, the literature on cognitive science gives reasons to make that assumption), the paradox does not need an explanation anymore.

Furthermore, it is obvious that the phenomena linked to paradoxes such as that studied in this paper can be very useful to evaluate alternative theories trying to explain human reasoning or offering different logical systems. However, the most interesting point is that it is also evident that there are options other than standard logic, coming from both ancient times and current psychological science, which appear to be able to account for certain problems related to the human inferential activity that this later logic cannot explain.

References

BARNES, J., BOBZIEN, S., & MIGNUCCI, M. Logic. In: ALGRA, K., BARNES, J., MANSFELD, J., & SCHOFIELD, M. (Eds.). *The Cambridge History of Hellenistic Philosophy*. Cambridge, UK: Cambridge University Press, 2008.

BRAINE, M. D. S. & O'BRIEN, D. P. (Eds.). *Mental Logic*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc., Publishers, 1998.

GOULD, J. B. *The Philosophy of Chrysippus*. Albany, NY: State University of New York Press, 1970.

HEMPEL, C. G. Studies in the logic of confirmation (I). *Mind, v.* 54, 1945, p. 1-26.

JOHNSON-LAIRD, P. N. Against logical form. *Psychologica Belgica, v.* 5, n. 3/4, 2010. p. 193-221.

_____. Inference with mental models. In: HOLYOAK, K. J: & MORRISON, R. G. (Eds.). *The Oxford Handbook of Thinking and Reasoning*. New York, NY: Oxford University Press, 2012.

_____. How to improve thinking. In: WEGERIF, R., LI, L., & KAUFMAN, J. C. (Eds.). *The Routledge International Handbook of Research on Teaching Thinking*. Abingdon, UK, & New York, NY: Routledge, 2015.

KHEMLANI, S., LOTSTEIN, M., TRAFTON, J. G., *et al.* Immediate inferences from quantified assertions. *The Quarterly Journal of Experimental Psychology, v.* 68, n. 10, 2015, p. 2073-2096.

LÓPEZ-ASTORGA, M. La paradoja de Hempel. *Ciencia Cognitiva, v.* 2, n. 3, 2008, p. 94-95.

_____. Are conditional and disjunction really comparable? *Universum, v.* 28, n. 2, 2013, p. 229-242.

_____. The conditional introduction rule and human reasoning: Findings from the mental models theory. *Cogency, v.* 6, n. 2, 2014, p. 29-37.

_____. Chrysippus' *indemonstrables* and mental logic. *Croatian Journal of Philosophy, v.* 15, n. 43, 2015, p. 1-15.

NICKERSON, R. S. Hempel's paradox and Wason's selection task: Logical and psychological puzzles of confirmation. *Thinking and Reasoning, n.* 2, 1996, p. 1-31.

OAKHILL, J. & GARNHAM, A. (Eds.). *Mental Models in Cognitive Science. Essays in Honour of Phil Johnson-Laird*. Hove, UK: Psychology Press, 1996.

O'BRIEN, D. P. Human reasoning includes a mental logic. *Behavioral and Brain Sciences, n.* 32, 2009, p. 96-97.

_____. Conditionals and disjunctions in mental-logic theory: A response to Liu and Chou (2012) and to López-Astorga (2013). *Universum, v.* 29, n. 2, 2014, p. 221-235.

_____. & LI, S. Mental logic theory: A paradigmatic case of empirical research on the language of thought and inferential role semantics. *Journal of Foreign Languages, v.* 36, n. 6, 2013, p. 27-41.

_____. & MANFRINATI, A. The mental logic theory of conditional proposition. In: OAKSFORD, M. & CHATER, N. (Eds.). *Cognition and conditionals:* Probability and Logic in Human Thinking. Oxford, UK: Oxford University Press, 2010.

O'TOOLE, R. R. & JENNINGS, R. E. The Megarians and the Stoics. In: GABBAY, M. D. & WOODS, J. (Eds.). *Handbook of the History of Logic.* Volume I. Greek, Indian and Arabic Logic. Amsterdam, The Netherlands: Elsevier, 2004.

SEDLEY, D. The negated conjunctions in Stoicism. *Elenchos, v.* 5, 1984, p. 311-317.

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