

## What does extensionality show in the *Tractatus*?

### ABSTRACT

Extensionality and extensionalism are common themes in Analytic Philosophy. The early Wittgenstein of the *Tractatus* is also taken to hold a thesis of extensionality. Extensionality in the *Tractatus* is associated with sentence 5, where Wittgenstein claims that a proposition is a truth-function of elementary propositions. The notion of a truth-function in the *Tractatus* is approached by an *operational view* in this paper that takes the truth-functions themselves as generated by a truth-operation. In this sense, the truth-operation is generating the notation itself, not an interpretation of some formal language. Extensionality in the *Tractatus* is approached in three steps, illustrating first what an operational reconstruction can show about extensionality, continuing with the role the *Tractatus* assigns to extensionality, and concluding by comparing it to other uses of the term extensionality in the Analytic tradition.

**Keywords:** Extensionality; Truth functionality; Early Wittgenstein; *Tractatus*.

### RESUMO

Extensionalidade e extensionalismo são temas comuns em Filosofia Analítica. O primeiro Wittgenstein, o do *Tractatus*, também é tomado como defendendo extensionalidade. Extensionalidade no *Tractatus* é associada à sentença 5, onde Wittgenstein reivindica que a proposição é uma função de verdade de proposições elementares. Neste sentido, a operação de verdade gera a notação ela mesma, e não uma interpretação para alguma linguagem formal. Extensionalidade no *Tractatus* é abordada em três passos, a saber, ilustrando, primeiramente, o que uma reconstrução operacional pode mostrar sobre extensionalidade. Em seguida, analisando o papel que o *Tractatus* atribui à extensionalidade, e concluindo ao comparar esta propriedade com outros usos do termo extensionalidade na tradição analítica.

**Palavras-chave:** Extensionalidade; Vero-funcionalidade; primeiro Wittgenstein; *Tractatus*.

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## Introduction

The aim of this paper is to discuss the theme of extensionality in Wittgenstein's *Tractatus Logico-Philosophicus*<sup>1</sup>. I will focus mainly on sentence 5 and its sub-sentences or commentaries according to the *Tractatus* numbering system. The goal is to reconstruct the particular variety of extensionality presented in the *Tractatus* by the early Wittgenstein and to contrast it to some extent with other notions of extensionality.

Sentence 5 of the *Tractatus* is commonly referred to as a *thesis of extensionality* (Carnap 1937, p. 188, Black, 1964) p. 219, Frascolla (2007) p. 118. Rosenberg (1968) p. 341). The *Tractatus* in its idiosyncratic style and composition lacks a clear argumentation structure and the goal is here to discuss in what way, if at all, extensionality should be taken as a thesis of the *Tractatus*. I argue that without connecting extensionality or its alleged thesis in sentence 5 to the operation  $N(\bar{\xi})$  and the various forms of notations used in the *Tractatus*, there can be no clear understanding of the notion of extensionality that is shown in the *Tractatus*.

My thesis is that a proper understanding of extensionality in the *Tractatus* linked to  $N(\bar{\xi})$  shows a demystification of logic. But that needs some qualification. The *Tractatus* does not fully expound what is now called first order logic. I do not aim to characterize the actual *Tractatus* logic in a formal way, but I retreat to the following formulation: the operation  $N(\bar{\xi})$  demystifies the particular logic endorsed in the *Tractatus*. The demystification that I use here refers to the final passages of the *Tractatus* leading up to its famous final call for silence about the unspeakable. The mystical in those final passages is associated with the *feeling* of the world as a limited whole (T 6.45) and the unspeakable (T 6.522).

In this paper, I will approach extensionality in the *Tractatus* in three steps. Firstly, I will consider what  $N(\bar{\xi})$  shows if it is taken as an operational device that generates a notation, rather than generating functions in their usual sense which is different from the one introduced in the *Tractatus*. Secondly, I will consider what role extensionality takes in the *Tractatus* according to Wittgenstein's own remarks.

In the third step I will reflect on extensionality in the *Tractatus* by comparing it to other uses of the term 'extensionality'. This will consist of one almost contemporary treatment explicitly mentioning Wittgenstein as the founder of the thesis of extensionality (Carnap's *Logical Syntax of Language*) and a recent retrospect on extensionality covering a long history of research into this topic (Quine's paper *Confessions of a Confirmed Extensionalist*).

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<sup>1</sup> From here on this work will be referred to as the *Tractatus*. Reference to parts of the work will be given by the abbreviation T followed by a number according to the numbering system Wittgenstein introduces himself or just by a number when the context makes it clear that I am talking about the *Tractatus*.

## The base for $N(\xi)$ : elementary propositions

One central motivation for the *Tractatus* is analysis. The existence of a unique and complete analysis is very clearly stated in sentence 3.25. However, in this remark there is a curious vagueness of *what* there is one and only one analysis of. In German, Wittgenstein uses a singular possessive construction of 'the proposition' ('des Satzes'). Analytic Philosophy takes Russell's analysis of definite descriptions and Frege's analysis both of identity and the natural number as its paradigm cases of analysis. From this perspective one would rather expect to have a unique analysis of many different propositions. But Wittgenstein does not carry out any particular analysis as explicitly as Frege or Russell. Rather, and this gives the thesis of extensionality its special character in this context, the *Tractatus* radically scrutinizes not only the composition of the analysandum, but also the composition of the notation used for logical analysis itself. Characteristically, it is when he is talking about logical grammar or language and their signs that Wittgenstein's remarks in the *Tractatus* can be read as commenting on other thinkers' work (T 3.325, T 3.331, T 3.332, T 4.431, T 5.452). In those relatively rare cases the *Tractatus* can be seen as participating in an argumentative discussion rather than relying on the confessional but certainly not conventional style as the hallmark of the *Tractatus*' composition.

The notion of the elementary proposition is central to the understanding of the role of extensionality in the *Tractatus*. Elementary propositions are taken to be the simplest expressions asserting the existence of a state of affair (T 4.21). Here, 'exists' should be read in the sense of the 'what is the case' formulations in sentence 2 and, in particular, sentence 1.12, that states that the totality of facts also determines what is not the case. Elementary propositions are further characterized by their logical independence (T 5.134) and the impossibility of two elementary propositions contradicting each other (T 4.211). The existence of elementary propositions is a precondition for logical construction, that is, logical complexity, because they form the basis of the operation  $N(\xi)$ . This time, 'existence' of 'the existence of elementary proposition' requires a different reading than the one above.<sup>2</sup> Wittgenstein shuns any example of an elementary proposition and does not give any indication about a class of particular sentences to constitute elementary propositions. Rather, the application of logic determines which elementary propositions exist (T 5.557). I take this to be a very important aspect of elementary propositions because it means that the characterization of elementary propositions cannot and should not be taken as a positive test for being an

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<sup>2</sup> The German text correspondence to this difference in meaning is the use of 'Bestehen' and the corresponding 'Nicht-Bestehen' in sentence 2 and sentence 1.12 in contrast to the use of 'es gibt' in sentence 5.557.

elementary proposition and that it is fruitless to discuss candidates for elementary propositions. This can also be taken as pointing towards an interpretation that takes elementary propositions not so much as an observation about sentences or propositions at all, but rather as showing something about logic. What logic needs before any application are simple units that reflect the complete generality and unspecificness of states of affairs, of either being the case or not being the case.

The elementary propositions take a peculiar middle position between the opening part of the *Tractatus* with their reflection on the basic distinction between being the case or not being the case and the *feelings* associated with the mystical of the final passages. According to both the Ogden- and Pears/McGuinness-translations in sentence 4.411, Wittgenstein declared that the understanding of the general sentences depends '*palpably*' on the elementary propositions. However, the German word Wittgenstein uses and emphasizes in italics is '*fühlbar*'. It can be *felt* that the understanding of any proposition depends on the elementary propositions.

The sign of elementary propositions does not stand outside the realm of states of affairs themselves. Further evidence for this line of thought is sentence 4.221, where Wittgenstein declares that the ultimate goal of analysis is elementary propositions. Beyond that, logical analysis cannot go any further. Further decomposition does not yield further propositions but only names. So one way to look at the lack of examples and the lack of a positive test for elementary propositions is to take elementary propositions as the destination of logical analysis and accept sentences of ordinary and scientific talk as already being logically complex. Thus, logical analysis is not done when a preferred reading or interpretation of an ordinary sentence is presented in a logical notation, but when it is broken down to components that no longer show logical dependence on each other.

Again, analysis in this sense is not carried out in the *Tractatus*. What I take to be the main objective of sentence 5 and its comments according to the numbering system is to consider the methods for analysis. This is done by keeping open any decision about where to start with analysis and turn the direction around by asking what a notation that is completely void of any predetermination of application would look like and what successful analysis would ultimately lead to. That a sustainable method of analysis is actually something worth looking for is important for the Tractarian conception of philosophy, since critical inquiry into language is one task left for philosophy (T 4.0031), apart from delimiting the realm of science (T 4.113). However, if the method used for this critical inquiry is itself based on unfounded distinctions and stipulations it may be best to remain silent. That the *Tractatus* is not silent on the issues of logical notations is ample evidence that there is hope, in contrast to the realms of aesthetics and ethics. Although these may be felt to

be more important for the problems of our lives (T 6.52), they are beyond what can be talked about.

What Wittgenstein says in the *Tractatus* in sentence 5 about propositions is that a proposition is a truth-function of elementary propositions. The following interpretation of sentence 5 and extensionality in the *Tractatus* is guided by the Tractarian warnings that truth-functions are not material functions (T 5.44) and that they are not to be confused with operations (T 5.25). Heeding these warnings is what I call an *operational view* on extensionality in the *Tractatus*. What can be learned positively about the notion of truth-functions in the Tractarian sense is that they can be listed for a given number of elementary propositions (T 5.1, T5.101) and that they are *generated* by truth-operators (5.3). Later in the *Tractatus* the truth-operators are limited to only one,  $N(\bar{\xi})$  (T 6.001). The next section focuses on how  $N(\bar{\xi})$  can be thought of as a means to generate truth-functions.

## Extensionality as operation on most general signs

It is in the sign of the elementary proposition and in the sign of the sentence in general constructed by the operation  $N(\bar{\xi})$  that mirroring facts becomes possible without taking a point-of-view beyond or outside the world, the totality of states of affairs. The composition of the sign of the elementary proposition and the logical forms of the sentences in the Tractarian notation become clear because we chose to *write* it so and *give* the component signs  $T$  and  $F$ . As such, they are different from ordinary language sentences not composed for showing logical form but for other communicative means (T 4.002).  $T$  and  $F$  are mere convenience. Only two restrictions are important: the manifold of the composition of the elementary proposition must match the manifold of the division of being the case or not being the case, and the complex sentence must in principle be able to assert agreement or disagreement to all possibilities of obtaining and not obtaining that the elementary propositions require of which they are composed. The second restriction yields the listing of truth-functions for the case of two elementary sentences. Finally, in order to count as meaningful and expressible, the mark of the sentence must still offer different truth-possibilities, which in the notational variant means the appearance of both  $T$  and  $F$  in its sign.

To investigate what the operation  $N(\bar{\xi})$  does for the *Tractatus'* view on logical notation, it is necessary to become acquainted with some of the instructions leading up to it. Some of these look like formulas or notation already, but by taking an *operational view*, I also mean to take these as abbreviations whose meanings are instructions on what to *do*. This procedure is textually supported in the *Tractatus*:

5.475 All that is required is that we should construct a system of signs with a particular number of dimensions—with a particular mathematical multiplicity.

5.476 It is clear that this is not a question of a number of primitive ideas that have to be signified, but rather of the expression of a rule.

The first case is the general term of a series of forms introduced in sentence 5.2522. Wittgenstein writes  $[\alpha, x, O'x]$ . He tells the reader that this is a variable but not that it is a sign or symbol. The first term is to be understood as the beginning of a series, the second term is any member arbitrarily selected, and the last term is its immediate successor according to the application of an operation.

Additionally, Wittgenstein introduces schemata of combinations of the truth-possibilities of elementary propositions (T 4.31). Again, these are *not* declared signs. In order to get to the sign of a sentence these schemes need to be written down again with an additional row like in the following example:

p	q	
T	T	T
F	T	T
T	F	
F	F	T

This whole is declared a propositional sign (T 4.442). It is given the shorthand  $(TTFT)(p,q)$ , but it is important to remember that this shorthand always depends on the scheme.

Finally, an operation is introduced in sentence 5.5:  $(- - - - T)(\xi, \dots)$ . This resembles the shorthand for the sentence sign above. It is given the shorthand  $N(\bar{\xi})$ . What that operation actually does, or rather, what one has to do to follow the rule expressed by  $N(\bar{\xi})$  is the topic of the following paragraphs. In sentence 5.5 Wittgenstein declares that it negates all propositions in the right-hand pair of brackets.

The general form of the truth-function which is the general form of the proposition as well combines these instructions in the variable  $[\bar{p}, \xi, N(\bar{\xi})]$ .

Focusing on sentence 6.001 it becomes necessary to understand how the construction of logical notation comes about and what the content of the thesis of extensionality is within the *Tractatus*. Wittgenstein declares that each sentence is the result of successive application of the operation  $N(\bar{\xi})$  on elementary propositions. He adds that with this, the transition from one sentence to another is also given (T 6.002).<sup>3</sup> But this cannot be taken as straightforward as

<sup>3</sup> I ignore the puzzling introduction of the operation  $\Omega'(\bar{\eta})$  as  $[\bar{\xi}, N(\bar{\xi})]'(\bar{\eta}) (= [\bar{\eta}, \bar{\xi}, N(\bar{\xi})])$ . It apparently defies what is introduced in sentence 5.2522 about formal series, and corner brackets with just two comma

it might seem, because successive application of  $N(\bar{\xi})$  to the result of a prior application does not yield the desired combinations of  $T$  and  $F$  as can be demonstrated for the case of two propositional variables for elementary propositions. The first suggestion from the *operational view* might be to exploit the resemblance of the operation  $(- - - - T)(\bar{\xi}, \dots)$  with the shorthand for propositional signs and assume that the application of  $N(\bar{\xi})$  consists of writing down the appropriate scheme for the number of propositions and filling in an  $F$  in the lowermost row. In shorthand:  $(TTTF)(p, q)$ . But just that would rule out *successive* application.

Additionally taking negation as switching the  $T$ s and  $F$ s used in *writing down* the schemes of elementary sentences we get the following progression:

- (1)  $N((TF)(p), (TF)(q)) = (FFFT)(p, q)$   $plq$
- (2)  $N((FFFT)(p, q)) = (TTTF)(p, q)$   $p \vee q$
- (3)  $N((TTTF)(p, q)) = (FFFT)(p, q)$   $plq$

Any further application of  $N(\bar{\xi})$  would obviously only reiterate this situation. Taking into account the application of  $N(\bar{\xi})$  to only a single elementary proposition we get this:

- (4)  $N((TF)(p)) = (FT)(p)$   $\sim p$
- (5)  $N((FT)(p)) = (TF)(p)$   $p$

Interestingly enough, this procedure yields two sets of functionally complete connectives from a modern perspective:  $\{I\}$  and  $\{\vee, \sim\}$ .<sup>4</sup> However, the point I intend to make is that the operation can be taken as instructions on how to construct all the listable truth-combinations of sentence 5.101 as truth-functions in the Tractarian sense, not by combination of connectives associated with one or some of the truth-functions in the modern sense of being a function from a set of truth-values to truth-values. If, in the case of two propositional values, the introduction of  $plq$  by the associated function  $\{F, F\} \rightarrow T$  and  $F$  otherwise, is taken as the whole point of  $N(\bar{\xi})$ , then the conception collapses back into a functional instead of an operational view about logical form in the *Tractatus*.

Another possibility is to take the schemes in sentence 4.31 themselves, take them as  $\bar{p}$  and successively operate on by taking neighboring rows as consecutive entries as  $\bar{\xi}$  and recording 'T' only in case two 'F's are each other's neighbor:

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separated entries are not used anywhere else in the *Tractatus*.

<sup>4</sup> There is a potential source of confusion here over the sign that is associated with the function  $(\{F, F\} \rightarrow T$  and  $F$  otherwise), by the functional view. In the *Tractatus*, Wittgenstein introduces the  $I$  as 'neither  $p$ , nor  $q$ ' (T5.1311). But the sign  $I$  is now called Sheffer stroke and more commonly associated with the function  $(\{W, W\} \rightarrow F$  and  $T$  otherwise). I will stick to the *Tractatus* convention when discussing  $I$  and take  $plq$  as a whole as a name for the truth function in the Tractarian sense  $(FFFT)(p, q)$ .

P	Q	(p,q) <sub>1</sub>	(p,q) <sub>2</sub>	(p,q) <sub>3</sub>	(p,q) <sub>4=1</sub>
T	T	F	F	T	F
F	T	F	F	T	F
T	F	F	T	F	F
F	F	T	F	F	T
$\bar{p}$		$\bar{\xi}_2$			
	$\bar{\xi}_1$		$\bar{\xi}_3$		

Again, the procedure terminates too early and does not achieve what  $N(\bar{\xi})$  is supposed to.

Another dead end is to start with the schemes and then allow recombination of any produced column with the signs of the elementary sentences in the proper manifold that is then determined by the fact that the other function  $(---)(p,q)$  already requires four entries of 'T' or 'F'. It is at least not enough in its own right, but it can be augmented to generate all truth-functions. The following short-hands of the generation start with the restriction of recombination with the elementary propositions which is afterwards lifted.

$$(6) N((TFTF)(p), (TTF)(q)) = (FFFT)(p,q) \quad (p,q)_1$$

This first step already calls for a pause, though. Wittgenstein repeatedly associates this first step of applying  $N(\bar{\xi})$  with the *Principia* notation ' $\sim p.\sim q$ ' (T 5.101, 5.1311). But in the proper notation it is revealed that neither  $\sim p$  nor  $\sim q$  are components in the proper understanding of complexity of the sentence sign:  $(FFFT)(p,q)$ . It is important that this is the *first* step in logical composition on the basis of two elementary propositions. There can be no other. It is possible that this result is reached again by more complex composition, but no other truth-function can be reached before that. What the composition of ' $\sim p.\sim q$ ' suggests is that there is a combination of two smaller parts which are in turn not simple. This is revealed to be misleading by Tractarian lights.

What is shown is that from the Tractarian perspective, the *Principia* notation actually suffers from the same deficits that natural language is ridden with. The way *this* notation is concatenated and treated makes it look as if there were more structure than there actually is. While it can be used to show that what seems to be the logical form of a proposition is not its actual one (Russell's merit, according to 4.0031), the notation itself does not live up to the standard of revealing instead of expressing the real logical form. That argument leaves a lacuna as to whether reformulation of the *Principia* system into one with *I* as the sole primitive would be acceptable from the viewpoint of the *Tractatus*. The remarks in 5.1311 do point in this direction. However, this cannot be addressed without evoking the requirements of autarky of logical matters and avoidance of hierarchy of logical propositions. It is not shown by the operative perspective on truth-combinations alone.

Successive application from (6) then yields:



- (7)  $N((FFFT)(p,q),(TFTF)(p)) = (FTFF)(p,q)$   $(p,q)_2$   
 (8)  $N((FFFT)(p,q),(TTTT)(q)) = (FFTF)(p,q)$   $(p,q)_3$   
 (9)  $N((FTFF)(p,q),(TFTF)(p)) = (FFFT)(p,q)$  (6)  
 (10)  $N((FTFF)(p,q),(TTTT)(q)) = (FFTT)(p,q)$   $(p,q)_4$   
 (11)  $N((FFTF)(p,q),(TFTF)(p)) = (FTFT)(p,q)$   $(p,q)_5$   
 (12)  $N((FFTF)(p,q),(TTTT)(q)) = (FFFT)(p,q)$  (6)  
 (13)  $N((FFTT)(p,q),(TFTF)(p)) = (FTFF)(p,q)$  (7)  
 (14)  $N((FFTT)(p,q),(TTTT)(q)) = (FFFF)(p,q)$   $(p,q)_6$   
 (15)  $N((FTFT)(p,q),(TFTF)(p)) = (FFFF)(p,q)$  (14)  
 (16)  $N((FTFT)(p,q),(TTTT)(q)) = (FFTF)(p,q)$  (8)  
 (17)  $N((FFFF)(p,q),(TFTF)(p)) = (FFFT)(p,q)$  (6)  
 (18)  $N((FFFF)(p,q),(TTTT)(q)) = (FFTT)(p,q)$  (10)

At this point, the operational procedures terminate. However, others can be generated by taking the ones produced and using them as single values for  $N(\bar{\xi})$ . This must exclude the initial columns of  $p$  and  $q$  though, else this would confuse  $N(p)$ , which is  $(FT)(p)$  and not  $(FTFT)(p)$ .

- (19)  $N((p,q)_1) = (TTTT)(p,q)$   $(p,q)_7$   
 (20)  $N((p,q)_2) = (TFTT)(p,q)$   $(p,q)_8$   
 (21)  $N((p,q)_3) = (TTFT)(p,q)$   $(p,q)_9$   
 (22)  $N((p,q)_4) = (TTFF)(p,q)$   $(p,q)_{10}$   
 (23)  $N((p,q)_5) = (TFTF)(p,q)$   $(p,q)_{11}$   
 (24)  $N((p,q)_6) = (TTTT)(p,q)$   $(p,q)_{12}$

Only four binary truth functions are missing. These can now be generated in numerous ways. One example for each are the following shorthands, again with the reservation that these should really be thought about as introduced above:

- (25)  $N((p,q)_3, (p,q)_5) = (TFFF)(p,q)$   $(p,q)_{13}$   
 (26)  $N((p,q)_{13}, (FTTT)(p,q))$   $(p,q)_{14}$   
 (27)  $N((p,q)_2, (p,q)_3) = (TFFT)(p,q)$   $(p,q)_{15}$   
 (28)  $N((p,q)_{15}, (FTTF)(p,q))$   $(p,q)_{16}$

The method must now conclude, because all 16 possible combinations of writing  $T$  and  $F$  have been passed through. This is not enough to demonstrate that this method works for all bases of elementary propositions and a demonstration of that cannot rely on functional completeness of the truth-function associated with the Sheffer stroke ( $\{F, F\} \rightarrow T$  and  $F$  otherwise). I do

not have such a demonstration, but there are some things that can be pointed out for the first application of  $N(\bar{\xi})$  for a base of three elementary propositions:

p	Q	R	$N(p,q,r)$
T	T	T	F
F	T	T	F
T	F	T	F
T	T	F	F
F	F	T	F
F	T	F	F
T	F	F	F
F	F	F	T

This makes it clear that, in principle, the method is still available to check each line and write  $T$  in each line there are only  $F$ s in the way it was introduced above. So basically, the notational practice of the *Tractatus* comes down to checking a line occurrence of the same entry. Which, significantly, is the same way we determine in this procedure both tautologies and contradictions but, in that case, by columns.

In the tautologies and in the contradictions the compositions of the symbol collapse because the manifold that is required for presenting a state of affairs is lost. This does not mean that there is no complexity in the sign (T 4.4661). It is important that it is not lost, for this keeps them open for application of the operation  $N(\bar{\xi})$ . Even though the logical sentences do not enable representation, they are still within the reach of the construction of the meaningful sentences by  $N(\bar{\xi})$ .

The Tractarian distinction of saying and showing has received enormous attention but some of the many entangled problems in this context of this distinction cannot be untangled unless writing or writing down as an action does not also receive some attention in the *Tractatus*. The logical form of the sentence is something that cannot be said as it must be shown. But the structure of the notation is not in this way beyond the reach of analysis. It shows itself immediately because we *choose* to set it up this way. In that, we are free to declare what the components of construction are and it is clear from the manipulation of signs, in the *Tractatus* the single truth operation  $N(\bar{\xi})$ , which are the units of this manipulations because they are pointed out by declaring them so. In this way, we fix the relevant parts of the state of affair that is the notation of a sentence and in particular, an elementary proposition.

### How $N(\bar{\xi})$ serves the *Tractatus* view on logical notation

In this part I will focus on what is claimed in the *Tractatus* about extensionality introduced by the generation of a notation designed to show its

logical form. From the more conventional perspective of modern logical expression and form, tutored by assuming distinction of syntax and semantics, it looks as though Wittgenstein supplied merely a method of truth tables as a semantics and the reduction of all truth table combinations to one operation. However, from the Tractarian perspective much more is at play, and I will present these under the general label of demystification of logic.

The first achievement from the Tractarian perspective is to eradicate the distinction between axioms and theorems in a logical system. This has several advantages from the Tractarian view on logic in general, and the laudation of this merit finds expression in several formulations.

One aspect of this is that composition by  $N(\bar{\xi})$  obliterates any hierarchy among logical sentences. Wittgenstein does not want to convince the readers of the *Tractatus* that there is a special class of logical sentences, the axioms, from which other logical sentences are derived. This would immediately call into question where these axioms come from, and why they are these and not others. Furthermore, this very procedure of setting up a logic would suggest that there are many sentences of logic that say different things, but all logical sentences say nothing, according to sentence 5.43.

This is closely connected to the demand for logic expressed in the *Tractatus* claiming logic must take care of itself (T 5.473). A procedure that takes a class of axioms to start with to produce more logical sentences needs some justification as to why these were logical to begin with. But that would have to be an external justification. The operation  $N(\bar{\xi})$ , in contrast, as the manifestation of logical composition, does take a start somewhere, either a selection of elementary sentences or possibly their indefinite whole, but these are just the simplest possible units of independent presentation of states of affairs. The fact that one can determine which logical constructions are logical sentences with this meager basis relying only on the reflection of the possibility of being the case and not being the case is important. In the context of the *Tractatus* the proper notation must make it possible to see them in the sign of the sentence (T 6.122).

With the operation  $N(\bar{\xi})$  it is meant to show that construction of a logical notation that 'takes care' of itself is not only free of outside justification but also reduction to the most simple procedure of discovering the logical sentences is possible. It is not the rules of inference that are most intuitive and simple suggesting more and more complex sentences of logic, but the procedures of checking rows and columns for the proper manifold. It is in this way that rules of inference in a proper notation would become superfluous (T 5.132).

Both requests, autarky and freedom of hierarchy reflect on a written notation as provisions to make analysis possible. Ordinary language is already in proper logical order (T 5.5563), but ordinary language is not meant to show the formal properties of their construction but is made for other communicative

means. Only a constructed language can show the formal properties (T 5.556). This construction, which is a doing, must be the most simple thing (T 5.4541). The operational approach presented as a construction of notation is just as simple as the very demand to distinguish between the possibility to obtain or not to obtain.

Another aspect of the demystification of logic in the *Tractatus* is that tautologies and contradictions receive equal treatment in the notation. In principle, this reflects that logic is not concerned with the truth or falsity of sentences. Since both the forms of tautologies and contradictions are set out by the same mechanism and show the same defect from the perspective of presentation, namely the lack of sufficient manifold in their notation.

In sentence 6.1202, Wittgenstein observes that contradictions might as well be put to the same use as the tautologies. Since logic needs to be put to all uses, contradictions serve as limits to meaningful notation just as much as tautologies do.

Finally, the operation  $N(\bar{\xi})$  and extensionality account for the logical connectives, particularly negation, the status of which is questioned for example in sentence 5.512:

5.512 'p' is true if '~p' is false. Therefore, in the proposition '~p' when it is true, 'p' is a false proposition. How can the stroke '~' make it agree with reality?

The answer in the *Tractatus* is that the stroke does not do anything. The only thing that can be the negation, which makes agreement with reality an option, is that which is common to all the variants in *this* notation that are equivalent with 'p': '~~~p',  $\sim p \vee \sim p$  which in turn is that no line of their associated truth function has anything in common with the sign of  $(WF)(p)$  which is the mark of the negative (of which there is only one) of a sentence to another (T 5.513). This is extended to all the connectives and declared a fundamental idea:

4.0312: My fundamental idea is that the 'logical constants' are not representatives; that there can be no representatives of the logic of facts.

In effect, in a proper notation, the signs of a logical notation just are most general facts that mirror states of affairs. And they are so, because they are made.

## Tolerance and rigor: Carnap and Quine on extensionality

The final step to approach extensionality in the *Tractatus* is to compare it with other cases where the term is used.

Carnap's *Logical Syntax of Language* mentions Wittgenstein and Russell as adhering to a thesis of extensionality (CARNAP, *Logical Syntax of Language*,

p. 245). He also cites his own *Logischer Aufbau der Welt*. What all these sources neglect, according to Carnap, is that there is not only one language but several. He aims this criticism particularly at Wittgenstein who is accused of using 'the language' with a definite article throughout the *Tractatus*.

We have seen that Wittgenstein maintains that there is only one final analysis and that there is the goal of finding the one notation that makes everything clear and easy. However, Wittgenstein acknowledges the possibility of different symbolic systems in sentence 4.5:

4.5 It now seems possible to give the most general propositional form: that is, to give a description of the propositions of any sign-language whatsoever in such a way that every possible sense can be expressed by a symbol satisfying the description, and every symbol satisfying the description can express a sense, provided that the meanings of the names are suitably chosen.

Wittgenstein adds that this description of the proposition of any such language can only be the most general form. What Wittgenstein aims at is on a different level than Carnap's thesis of extensionality which is a claim about the translatability of different languages into each other. Wittgenstein is trying to show something about the possibility of constructing any notation or sign language, and that requires not only making the distinction of being the case or not being the case, but also at most that distinction. The translation of languages must rely on the claim that at least something is the case: some parts of these languages correspond to each other. But this makes it clear that this form of extensionality already presupposes the one Wittgenstein has in mind.

Carnap takes the possibility of non-extensional languages seriously and retreats to a statement he considers more cautious than the thesis of extensionality associated with sentence 5 in the *Tractatus*, the claim that all propositions are truth-functions of elementary propositions.

The aim of *Logical Syntax* is very similar to what has been claimed about the motivation of the *Tractatus*. Carnap claims that the scientific work of the philosopher is logical analysis (CARNAP, 1937, p. 13). However, Carnap's approach is to give a method for talking about the sentences of logic and to express the exact manner the findings of logical analysis. While Wittgenstein is interested in letting logical notation show its formal properties from within, Carnap takes the route of formulating a metalogic.

Finally, Tractarian extensionality is similar yet subtly different from the doctrine of extensionalism Quine espoused throughout his career. I shall take Quine's self-commenting and self-summary of *Confessions of a Confirmed Extensionalist* as a convenient way to look at some of the many aspects that are linked to extensionality in Quine's thinking. Famously, and as Quine points out himself in the opening paragraph, extensionalism has been a phi-

philosophical doctrine or *policy* that Quine held on through his whole career. Extensionalism is declared “a predilection for extensional theories.” (QUINE, 2004, p. 329). But more important than this is the definition of extensionality itself: [A]n expression is extensional if replacement of its component expressions by coextensive expressions always yields a coextensive whole (QUINE, 2009).

There is a similar focus on notation, particularly in the notion of *semantic ascent*, the “strategy to talk about expressions” (QUINE, 2004, p. 337) instead of talk about things with unclear identity conditions, like properties or meaning. In this way, there appears to be similarity between Quine and Wittgenstein’s approach to logical notation. However, in getting at the notion of coextensive expressions we see how different the approaches in fact are. Quine discusses the coextension of three sorts of expressions: closed sentences; predicates, general terms, and open sentences; as well as singular terms. Much to the point, what predicates, open sentences, and general terms are only comes together once there is already a logical theory: “They are what the open letters in quantification theory stand for. Open sentences are the most graphic of the three renderings.” (QUINE, 2004, *Ibid.*, p. 329). Unlike Wittgenstein’s Tractarian conviction that proper logical notation must be guided by the consideration of extensionality that is itself manifested in the signs of the logical notation, for Quine logical notation comes first.

As to the usefulness of extensionality Quine cites the ‘clarity and convenience’ that come with the possibility of interchanging coextensive components *salva veritate*. This in turn gets so much emphasize that Quine adds: “I doubt that I have ever fully understood anything that I could not explain in extensional language” (QUINE, 2004, p. 331).

This does, in a way, put Quine’s view on extensionality closer to Wittgenstein’s than Carnap’s. Carnap’s version of the thesis of extensionality that accepts intensional languages by their own right but claims they are open for translation into an extensional language presupposes that those intensional languages can be understood. Both Quine and Wittgenstein take the stand that outside the extensional, there is nothing to understand.

The motivations for both Quine and Wittgenstein were apparently also very close. We see Wittgenstein complaining about the use of ‘words’ in the introduction of definitions and basic laws of *Principia* in sentence 5.452 and, likewise, Quine describes what bounded his admiration for the *Principia*:

My admiration was not quite unbounded. It was bound by the explanations in prose that were preposed and interposed as explanatory chapters and in briefer bits among the expanses of symbols (QUINE, 2004, p. 332).

It is also clear that there is a similar concern about the primitives of the foundations used in the notational language. Quine describes his two stages

of improving the *Principia* by using first individuals, classes and sequences in his dissertation, and finally just class inclusion and class abstraction in 1937, instead of propositional functions. Quine holds propositional functions of the *Principia* to be identifiable with propositions in the case of application to one variable, and to relations in the case of more variables. However, their lack of a principle of individuation makes them unclear.

Finally, then, one can say that even though the motivations for 'extensional' reform of logical notation in both Wittgenstein and Quine are similar, their ultimate concern is different. Wittgenstein wants an operation that with its clarity and simplicity makes the puzzling questions about notation vanish, while Quine constructs a formal foundation that adheres to the standard of 'no entity without identity'. For Quine, extensionality expresses the standard of clarity and simplicity in interchangeability and identity, questions about what there is. For the early Wittgenstein, it is the simplicity of applying a most simple rule to follow, a question about what to do.

## References

BLACK, M. *A Companion to Wittgenstein's 'Tractatus'*, first published. Cambridge: University Press, 1964, reprinted Ithaca: Cornell University Press, 1992.

CARNAP, R. *Logical Syntax of Language*, first published by P. Kegan, translated by A. Smeaton, London: Trench, Trubner & Co., 1937, reprinted London: Routledge, 2001.

FRASCOLLA P. *Understanding Wittgenstein's Tractatus*. London: Routledge, 2007.

QUINE, W.V. "Confessions of a Confirmed Extensionalist." In: FLOYD, J.; S. Shieh. *Future Pasts*. New York: Oxford University Press, 2001, reprinted in *Quintessence: basic readings from the philosophy of W.V.* Edited by R. F. Gibson, Jr., 329 – 337. Cambridge (MA), London: The Belknap Press of Harvard University Press, 2004.

ROSENBERG, J. F. "Intentionality and Self in the Tractatus." *Noûs*, v. 2, n. 4 1968, p. 341 – 358.

WITTGENSTEIN, L. *Tractatus Logico-Philosophicus*, translated from the German by C.K. Ogden, with an Introduction by Bertrand Russell. London: Boston and Henley: Routledge & Kegan Paul 1981. Online: K.C. Clement: Side-by-side-by-side edition, version 0.42 (January 05, 2015), containing the original German, alongside both the Ogden/Ramsey, and Pears/McGuinness English translations., <http://people.umass.edu/phil335-klement-2/tlp/tlp.html>.