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Buridan's logic: testability and models

A lógica de Buridan: testabilidade e modelos

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ABSTRACT

The theory of modality Buridan develops is linked to different propositions. This paper addresses one of those propositions. The aim is to show that, if the operator of necessity included in it is ignored, the proposition allows deriving, within first-order predicate logic, a reduction sentence with the characteristics required in Carnap's framework. Besides, the paper tries to argue that the mentioned operator of necessity can be understood not only in a technical sense (as modal logic does), but also in the way a naïve individual (without modal logical training) could interpret it. These latter arguments are given from the theory of mental models.

Keywords: Buridan. Carnap. Mental models. Necessity. Reduction.

RESUMO

A teoria da modalidade que Buridan desenvolve está vinculada com diferentes proposições. Este trabalho aborda uma dessas proposições. O objetivo é mostrar que, se o operador de necessidade incluído nela é ignorado, a proposição permite derivar, dentro da lógica de predicados de primeira ordem, uma sentença de redução com as características requeridas no *framework* de Carnap. Ademais, o artigo tenta argumentar que o mencionado operador de necessidade pode ser compreendido não apenas em um sentido técnico (como a lógica modal faz), mas também do modo como um indivíduo inocente (sem treinamento em lógica modal) poderia interpretá-lo. Estes últimos argumentos são dados a partir da teoria dos modelos mentais.

Palavras-chave: Buridan. Carnap. Modelos mentais. Necessidade. Redução.

Introduction

Based on works such as those of Hodges and Johnston (2017), Johnston (2015), and Read (2015), Dagys, Giedra and Pabijutaitė (2022) offer a number of propositions related to the theory of modal syllogism Buridan proposes. One of those propositions is interesting for this paper. That proposition is derived from the framework presented by Hodges and Johnston (2017) and Johnston (2015). However, the derivation also considers Buridan's works such as *Treatise on Consequences* and *Quaestiones in Analytica Priora*. These two works by Buridan are important because, following Dagys *et. al.* (2022), provide the concept of *necessitas conditionalis* ('conditional necessity'). Taking this concept into account, Dagys *et. al.* (2022) modify some propositions Johnston points out replacing conjunctions with conditionals. The relevant proposition here is one of those modified propositions. It is (1) (see DAGYS *et. al.,* 2022, p. 39).

(1) $\forall x\ [(Ox \wedge Sx) \rightarrow N(Ox \rightarrow Px)] \wedge \exists x\ (Ox\ \ Sx)$

Although some symbols are different here, proposition (1) is in Dagys *et. al.* (2022, p. 39). In it, '$\forall$' is the universal quantifier, '' represents the existential quantifier, '$\wedge$' stands for conjunction, '$\rightarrow$' denotes the conditional, and 'N' is the modal operator of necessity. Regarding 'O', 'S', and 'P', they are predicates. 'Ox' comes from Hodges and Johnston (2017), and it expresses that x exists 'at a world', 'Sx' indicates that x has the property to which the subject of the proposition refers, and 'Px' establishes that x has the property corresponding to the predicate of the proposition. Dagys *et. al.* (2022) also give a version of (1) including appearances of the modal operator of possibility. However, those appearances are removed in (1) in order to consider the *quod est* restriction. According to that restriction, the subject needs to be real (for further information on (1), see (DAGYS *et. al.*, 2022).

The reason why (1) is important is that, if operator N is removed, it can be transformed, just following first-order predicate calculus, into a reduction sentence of the kind Carnap (1936; 1937) proposes. To show this is the main aim of the present paper. The first section will explain what reduction sentences are within Carnap's (1936; 1937) framework. The second section will account for how (1) can lead to a reduction sentence if N is not considered. The next sections will argue that to ignore the presence of N do not lead to negative consequences. As Dagys *et. al.* (2022) mention, the relation between Buridan's philosophy and possible world semantics in modal logic is not evident. So, beyond the correspondences between Buridan's logic and first-order predicate logic that are to be found in the literature, it can be claimed that N should be interpreted as naïve people (i.e., people without modal logical training) understand necessity. To make this latter point, the paper will follow the theory of mental models (e.g., KHEMLANI; BYRNE; JOHNSON-LAIRD, 2018).

What reduction sentences are

Reduction sentences are described in Carnap (1936). They are sentences with the formal structure of (2) (the present paper will resort to the symbols used to express Carnap's reduction sentences in works such as LÓPEZ-ASTORGA, 2022a).

(2) $\forall x\ [Px \rightarrow (Qx \rightarrow Rx)]$

'P', 'Q', and 'R' are predicates in sentence (2) (which appears in LÓPEZ-ASTORGA, 2022a, p. 110). By virtue of its structure, (2) can be a reduction sentence. Within Carnap's (1936; 1937) approach, reduction sentences are applied to build scientific definitions assigning properties to those scientific definitions. In this particular case, (2) would be a reduction sentence for its last predicate, that is, R, and it would allow linking properties P and Q to R.

Nevertheless, if the properties indicated in the antecedents of the conditionals were unreal and, therefore, impossible for any being that could be thought, (2) would be always banally true. To avoid situations of this kind, Carnap (1936) also proposes a condition that sentences such as (2) must fulfill to be reduction sentences. Formula (3) (which follows the symbols in works such as LÓPEZ-ASTORGA, 2022a, too) expresses that condition.

(3) $\neg\forall x \neg(Px \wedge Qx)$

Symbol '$\neg$' means negation in formula (3) (which appears in LÓPEZ-ASTORGA, 2022a, p. 110). It guarantees that the mentioned problem does not happen. If (3) holds, cases of P and Q are not impossible. A reduction sentence with the structure of (2) and coherent with (3) can be deduced from (1) in first-order predicate logic. The next section addresses that deduction.

A reduction sentence deductible from Buridan's framework

Actually, the reduction sentence cannot be derived from (1), but from a formula akin to (1) without including the operator of necessity. That formula is (4).

(4) $\forall x\, [(Ox \wedge Sx) \rightarrow (Ox \rightarrow Px)] \wedge \exists x\, (Ox \;\; Sx)$

In first-order predicate calculus, (4) can be divided into (5) and (6).

(5) $\forall x\, [(Ox \wedge Sx) \rightarrow (Ox \rightarrow Px)]$

(6) $\exists x\, (Ox \wedge Sx)$

From (5), (7) can be inferred.

(7) $\forall x\, [Ox \rightarrow (Sx \rightarrow Px)]$

On the other hand, (6) is equivalent to (8).

(8) $\neg\forall x \neg(Ox \wedge Sx)$

Because (7) has the same structure as (2), it is possible to think that (7) is a reduction sentence. This is confirmed by virtue of (8), which has the same structure as (3). Therefore, (8) reveals that (7) fulfills the condition reduction sentences must fulfill.

The present paper is not claiming that Buridan anticipated reduction sentences or that Buridan's framework is an antecedent of Carnap's (1936; 1937) proposal. The only point the paper is making is that Buridan's theory of modality has the machinery necessary to deduce reduction sentences. Although the inferences Buridan's modal theory allows making seem to be consistent with system T, that does not justify to assume an obvious relation between

Buridan's philosophy and current semantics in modal logic (*DAGYS et. al.,* 2022). Likewise, although reduction sentences can be derived from Buridan's approach, that does not lead to an evident link between Buridan's logic and Carnap's philosophy of science.

A problem this argument needs to solve is that a reduction sentence such as (7) cannot be directly deduced from a proposition attributable to Buridan. (7) can be inferred from (4), not from (1). However, if there is no clear relation between Buridan's theory and possible world semantics, this problem can be easy to eliminate.

Necessity in the theory of mental models

If links between Buridan's theory of modality and current modal logic are not assumed, one might think that necessity in (1) should be interpreted as naïve individuals do, and not as contemporary logicians do. In other words, one might think that Buridan understood necessity as an average person, and not as modal logic does nowadays. So, the question would be how a naïve individual considers necessity.

The theory of mental models gives an answer. According to this theory, human reasoning is essentially modal in a natural way. Nevertheless, this modality has nothing to do with the current systems of modal logic (e.g., KHEMLANI; HINTERECKER; JOHNSON-LAIRD, 2017). The theory of mental models deals with several connectives in natural language (see also, e.g., JOHNSON-LAIRD; RAGNI, 2019). But the important connective here is the conditional. When the human mind processes a conditional, it takes a conjunction with three possibilities into account (see also, e.g., ESPINO; BYRNE; JOHNSON-LAIRD, 2020). Given, for example, (9),

(9) If A then B

Its 'conjunction of possibilities' is (10) (which follows the way to express the models or possibilities in works such as KHEMLANI *et. al.,* 2018).

(10) possible (A & B)
& possible (not A & B)
& possible (not A & not B)

Conjunction of possibilities (10) appears in Khemlani et al. (2018, p. 1890). It shows three possibilities seeming rows in a truth table in classical logic. But this view is not correct (e.g., JOHNSON-LAIRD; RAGNI, 2019). There are cases in which the possibilities are not those in (10) (see also, e.g., QUELHAS; RASGA; JOHNSON-LAIRD, 2017). For instance, the possibilities assignable to (11) are not similar to those in (10), but those in (12).

(11) If I read a book, I read this book

(12) possible (I read a book & I read this book)
& possible (I do not read a book & I do not read this book)

By virtue of its content, (11) is not a conditional; it is a biconditional. This is because, if I read this book, I read a book. Hence, a possibility such as the second one in (10) cannot be admitted.

Another example can be (13).

(13) "If there is gravity (which there is) then your apples may fall" (JOHNSON-LAIRD; BYRNE, 2002, p. 663).

Now, given that gravity exists, the possibilities are (see JOHNSON-LAIRD; BYRNE, 2002, where (13) is deemed as an example of 'strengthen antecedent' sentence):

(14) possible (there is gravity & your apples fall)
& possible (there is gravity & your apples do not fall)

That your apples fall is just a possibility. For that reason, they may fall, as in the first possibility in (14), or they may not fall, as in the second possibility in (14). What is known for sure is that there is gravity, which is the case in the two possibilities in (14).

On this basis, the theory of mental models can explain how people understand necessity and possibility:

> A clause can be evaluated as *possible* if it is affirmed in at least one possibility of the conjunctive set. It can be evaluated as *necessary* if it can be affirmed in all possibilities. And it is deemed *factual* if it is affirmed in a set of only one possibility (KHEMLANI *et. al.*, 2017, p. 665; italics in text).

In the case of (9) and (10), both A and B are possible. There is a case of A (first possibility) and two cases of B (second a third possibilities). Neither A nor B are in all of the possibilities.

Something similar occurs with (11) and (12). It is possible that I read a book and it is possible that I read this book. Both of them are in one possibility (the first one). However, none of them is necessary, as the other possibility (the second one) is that neither I read a book nor I read this book.

The situation is different in the case of (13) and (14). That your apples fall keeps being a possibility: in one possibility (the first one) they fall, and in the other possibility (the second one) they do not fall. Nevertheless, that there is gravity is necessary, since that is true in each of the possibilities.

These definitions of necessity and possibility allow checking that N in (1) is not a problem. The next section tries to show that.

Buridan, necessity, and reduction sentences

If one assumes that the theory of mental models is correct, and, accordingly, consistent with the way human beings reason, it can be said that the first conjunct of a sentence such as (4), that is, (5), refers to conjunction of possibilities (15).

(15) possible [(Ox $\wedge$ Sx) & (Ox $\rightarrow$ Px)]
& possible [not (Ox $\wedge$ Sx) & (Ox $\rightarrow$ Px)]
& possible [not (Ox $\wedge$ Sx) & not (Ox $\rightarrow$ Px)]

Conjunction of possibilities (15) reveals that Ox $\rightarrow$ Px is not, as it is in (1), necessary. It is true in two possibilities (the first and second possibilities). But in the last one (third possibility) it is false. However, the second conjunct in (4), that is, (6), helps understand what can happen when the antecedent of (5) is the case. In those circumstances, the possibilities in which the antecedent is not true, that is, the second and third possibilities in (15), are removed. The result is only one possibility: (16).

(16) possible [$(Ox \wedge Sx)$ & $(Ox \rightarrow Px)$]

Following the theory of mental models, (16) is a fact, since it includes just one possibility. Nevertheless, in that situation, $Ox \rightarrow Px$ is necessary, as it is in all of the possibilities (there is only one possibility, and $Ox \rightarrow Px$ is in that possibility).

This would be the natural manner to understand the necessity of $Ox \rightarrow Px$ according to the theory of mental models. It enables to see the correspondence between (1) and (4): in the natural sense, and not in the technical sense (i.e., not in the sense of possible world semantics and current modal logic), the consequent of (5) can be deemed necessary if (6) is the case. Thus, a transitive relation can be provided for the pairs (1) and (4), (4) and (7), and (1) and (7). If, based on the theory of mental models, in a natural and not technical way, (1) correspond to (4), and (4) allows deriving (7), in a natural and not technical way too, (1) should allow deducing (7).

Two more points with regard to this argumentation can be indicated. On the one hand, the account of the theory of mental models can be even simpler. People do not always consider all of the possibilities of the conjunction of possibilities corresponding to conditionals. That requires analytic reasoning and concentration. If reasoning is intuitive and quick, only the most evident possibility is detected (see also, e.g., BYRNE; JOHNSON-LAIRD, 2020). That possibility is that in which the two clauses happen, that is, the first one in (10), the first one in (12), the first one in (14), and the first one in (15), which is equivalent to (16). If this is right and individuals only take (16) into account when they process a sentence such as (5), as explained above, the consequent of (5) is necessary. This is because it is in all of the possibilities (there is just one possibility and the consequent is true in it).

On the other hand, given that the concept of reduction sentence is established by Carnap, one might think that the link between (1) and the reduction sentence that can be derived from it should consider the way Carnap understands necessity. The problem is that necessity for Carnap (1947) is akin to what it is in current modal logic and possible world semantics. Carnap (1947) does not resort to the phrasing 'possible world'. The expression Carnap prefers is 'state-description'. But a state-description is the same as a possible world. Hence, this would force to take possible world semantics into account in the analysis of (1).

If there were obvious links between Buridan's modality and possible world semantics, this objection could be overcome from a technical point of view. As it is known, Carnap's (1947) proposal is compatible with system S5, and S5 includes, among other axioms, axiom T. Universally quantified, axiom T can be expressed as formula (17).

(17) $\forall x\ (NPx \rightarrow Px)$

And (7) can be derived from (1) and (17). Thereby, because (8) can also be inferred from (1), (7) would keep being a reduction sentence (for relations between Carnap's framework of modal logic and the theory of mental models, see, e.g., LÓPEZ-ASTORGA, 2022b). Furthermore, axiom T allows interpreting Buridan's modal theory from system T (DAGYS *et. al.,* 2022).

However, given that it is not clear that Buridan's logic is linked to possible world semantics (DAGYS *et. al.,* 2022), it should be thought that necessity is understood by Buridan as naïve people understand it in a natural way. If it is assumed that the explanation from the theory of mental models about the natural manner to understand necessity is right, the account above suffices to ignore symbol N in (1) and consider just (4).

Conclusions

While the inferences that can be made within Buridan's semantics are compatible with system T, that does not imply that Buridan anticipated modal logic in the way it is considered nowadays (DAGYS *et. al.,* 2022). Nevertheless, that shows the potential Buridan's philosophy has. Likewise, while a reduction sentence can be deduced, based on first-order predicate logic, from a proposition linked to Buridan's framework, that does not mean that Buridan anticipated Carnap's analyses of science, or the idea of building a language for science as exact as possible. Nonetheless, this also reveals the potential of Buridan's logic.

The only difficulty with regard to the derivation of the reduction sentence following only first-order predicate calculus is that the initial proposition includes the operator of necessity. But, if Buridan's system is not related to possible world semantics, it can be thought that Buridan understood necessity as naïve people do. The question is, hence, how a naïve individual understands necessity. An option is to assume that naïve people consider necessity as the theory of mental models claims. If this is accepted, the difficulty disappears and the reduction sentence is not hard to infer.

So, at least two points deserve to be researched. On the one hand, it is required to keep working in order to check to what extent the theory of mental models describes what the human mind does. If the theory of mental models gets further support, the present paper will get further support as well. On the other hand, it is also relevant to continue to analyze Buridan´s theory of modality. The literature reveals that system T can capture that theory. This paper has also proposed that very theory can be addressed from the testability processes Carnap describes. Accordingly, it is worth reviewing in more detail more components of Buridan's framework in order to determine whether or not similar situations with regard to other later approaches are possible.

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